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MASS SHIFT OF AXION IN MAGNETIC FIELD

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A mass-shift of the axion propagating in an external constant homogenious magnetic field is calculated. The contributions via an electron loop and a virtual photon are examined. It is shown that the virtual photon contribution dominates substantially over the electron-loop one. Under the conditions of the early Universe the electron-loop contribution to the massless axion mass-shift is equal to zero while the virtual photon contribution is finite and can be of order of the recent restrictions on the axion mass.

The axion [1,2] obtains the mass due to the mixing with π^0-meson and, as a consequence, the Peccei-Quinn scale, f_a, is related to the axion mass, m_a, by the relation [3]: $m_a \simeq m_\pi f_\pi / f_a$, where m_π and f_π are the mass and the decay constant of π^0-meson. In models of an "invisible" axion [4,5] the axion mass is, in principle, arbitrary, however astrophysical and cosmological considerations provide an upper and lower bounds [3]:

$$10^{-6} \text{ eV} \lesssim m_a \lesssim 10^{-3} \text{ eV}. \tag{1}$$

The processes with weakly interacting particles, and with axions, in particular, are of importance under extreme external conditions which can be realized, in the early Universe as well as in astrophysical objects such as a magnetized neutron star or a supernova explosion. In studying of processes under such conditions one has to take into account non-trivial dispersions of particles. In considering axion processes the changing of the axion dispersion can occur substantial and, hence, should be investigated. In addition to the contribution to the axion self-energy via the electron loop, the other contribution via a virtual photon exists due to an effective axion-photon interaction in the external electromagnetic field. In this paper we show the importance of the photon-induced mass-shift of the axion in the strong magnetic field.

The contribution to the axion mass squared, δm_a^2, is connected with the real part of the field-induced amplitude ΔM of $a \to a$ transition by the relation:

$$\delta m_a^2 = -\mathrm{Re}\,\Delta M. \tag{2}$$

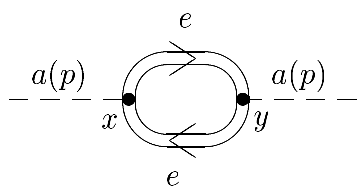

Figure 1. The external-field diagram of $a \to a$ transition via an electron loop.

In the second order of the perturbation theory $a \to a$ transition amplitude induced by the electron loop is described by the diagram shown in Fig. 1. Taking into account the external field influence means that one has to use exact electron propagators in the field which are drawn as double solid lines in Fig. 1. Below we consider the limit of the strong magnetic field when the field strength is the largest scale parameter ($|eB| \gg q_\perp^2, q_\parallel^2, m_e^2$, where m_e and e are the mass and electric charge of the electron, q_μ is an axion four-momentum, $q_\parallel^2 = q^2 + q_\perp^2$,[a] and $q_\perp^2$ is the squared axion momentum component orthogonal to the magnetic field strength $\mathbf{B}$). In this limit the electron-loop contribution, $\left(\delta m_a^2\right)_e$, has a form [7]:

$$\left(\delta m_a^2\right)_e \simeq -\frac{g_{ae}^2\,|eB|}{2\pi^2}\,\exp\left(-\frac{q_\perp^2}{2|eB|}\right)\,F\left(\frac{4m_e^2}{q_\parallel^2}\right). \tag{3}$$

Here, $g_{ae} = C_e m_e / f_a$ is an axion-electron coupling, C_e is a model-dependent parameter of order of unity, and the function $F(z)$ is:

$$F(z) = \begin{cases} \dfrac{1}{2\sqrt{1-z}}\left[\ln\left|\dfrac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right| - i\pi\Theta(z)\Theta(1-z)\right], & z < 1, \\ \dfrac{1}{\sqrt{z-1}}\,\arctan\dfrac{1}{\sqrt{z-1}}, & z \ge 1, \end{cases} \tag{4}$$

where $\Theta(z)$ is the unit step-function. The imaginary part of $F(z)$ in the kinematical region $0 < z < 1$ means that in the magnetic field the axion decay into an electron-positron pair $a \to e^+e^-$ [8] is allowed under the condition $q_\parallel^2 > 4m_e^2$.

The other contribution to the axion mass squared in the external field via the virtual photon is described by the diagram shown in Fig 2. We note that it is necessary to take into account the influence of the field on both the

[a] In calculations we use the metrics $g_{\mu\nu} = diag(1, -1, -1, -1)$, so that for any four-vector $a^2 = a_0^2 - \mathbf{a}^2$.

Figure 2. The external-field diagram of $a \to a$ transition via a virtual photon.

axion-photon coupling and the photon propagator. The photon contribution to the field-induced axion mass squared has a form [7]:

$$\left(\delta m_a^2\right)_\gamma = \bar{g}_{a\gamma}^2 B^2 q_\parallel^2 \,\mathrm{Re}\left(q^2 - æ^{(2)}\right)^{-1} . \tag{5}$$

Here, $\bar{g}_{a\gamma} = \alpha\bar{\xi}/2\pi f_a$ is an effective axion-photon coupling, α is the fine-structure constant, $\bar{\xi}$ is a model-dependent parameter of order of unity with taking into account the external field corrections [9], and $æ^{(2)}$ is an eigenvalue of the polarization operator corresponding the "second" photon mode in the notation of Ref. 6:

$$æ^{(2)} \simeq -\frac{2\alpha|eB|}{\pi}\left[\frac{4m_e^2}{q_\parallel^2} F\left(\frac{4m^2}{q_\parallel^2}\right) - 1\right], \tag{6}$$

where the function $F(z)$ is defined in Eq. (4).

The numerical estimations of the field-induced contributions to the axion mass-shift (the self-energy in the axion rest frame) are [7]:

$$(\delta m_a^2)_e \simeq -3.3 \times 10^{-31}\,\mathrm{eV}^2 \times C_e^2\,\frac{B}{B_0}\left(\frac{10^8\,\mathrm{GeV}}{f_a}\right)^2\left(\frac{m_a}{10^{-3}\,\mathrm{eV}}\right)^2, \tag{7}$$

$$(\delta m_a^2)_\gamma \simeq 1.26 \times 10^{-15}\,\mathrm{eV}^2 \times \xi^2\left(\frac{B}{B_0}\right)^2\left(\frac{10^8\,\mathrm{GeV}}{f_a}\right)^2, \tag{8}$$

where $B_0 = m_e^2/|e| = 4.41 \times 10^{13}$ G is the so-called Schwinger value. For the magnetic fields strength of order of the Schwinger value the long-range contribution via the virtual photon is 10^{16} times larger then the electron-loop one but both contributions are very small. It means that in studying axion processes in magnetized astrophysical objects with the field strength $B \sim 10^{13} - 10^{15}$ G one can neglect the field influence on the axion mass (1).

However the situation is possible when the field-induced contribution to the axion mass-shift is essential. It is the case when the axion is a massless particle as, for example, before the QCD phase transition under the conditions of the early Universe. Note that at this stage of the Universe evolution a very strong magnetic field with the strength $B \sim 10^{22} - 10^{23}$ G can exist [10]. In this

case the electron-loop contribution to the axion self-energy, $(\delta m_a^2)_e$, which is proportional to the axion transfer momentum, is equal to zero in the axion rest frame while the virtual photon one is not vanish [7]:

$$\delta m_a \simeq 0.058\,\mathrm{eV} \times \xi \left(\frac{10^8\,\mathrm{GeV}}{f_a} \right) \left(\frac{B}{10^{23}\,\mathrm{G}} \right)^{1/2} . \tag{9}$$

It is seen that under the early Universe conditions the massless axion acquires a mass due to the long-range interaction with photons. As the model-dependent parameter ξ is typically of order of unity [3], the axion mass induced by a primordial magnetic field can be as large as its recent value (1).

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